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by aid of Figs. 5

ices, use Eq. (3)
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example 4).

example, suppose
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What is the mis-
and the line?
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els. Then, using
be 2.22. On the

$R_0| \ll 1$

ITT "Reference Data for Radio Engineers"

Smith chart, locate the point corresponding to 0.35 wavelength toward the generator from a voltage maximum, and swr=2.22. Read the input normalized impedance as $0.62+j0.53$ with respect to $Z_0=50$ ohms. Now the mismatch loss at the input can be determined by use of (3). However, since the generator impedance is nonreactive, (1) can be used if desired. Refer to the following paragraph and to the "Notes on Equation (3)" above.

With respect to $100+j0$ ohms, the normalized impedance at the line input is $0.31+j0.265$, which gives swr=3.5 according to the Smith chart. Then by (1), $P_m/P=1.45$, giving a mismatch loss of 1.62 decibels. The transducer loss is found by using the results of examples 3 and 4 in (4). This is

$$1.27+2.00+1.62=4.9 \text{ decibels.}$$

ATTENUATION AND RESISTANCE OF TRANSMISSION LINES AT ULTRA-HIGH FREQUENCIES

The normal or matched-line attenuation in decibels/100 feet is

$$A_{100}=4.34R_t/Z_0+2.78f\epsilon^{1/2}F_p$$

where the total line resistance/100 feet (for perfect surface conditions of the conductors) is, for copper coaxial line

$$R_t=0.1(1/d+1/D)f^{1/2}$$

and for copper 2-wire open line

$$R_t=(0.2/d)f^{1/2}$$

where D= diameter of inner surface of outer coaxial conductor in inches, d= diameter of conductors (coaxial-line center conductor) in inches, f= frequency in megahertz, ϵ= dielectric constant relative to air, and F_p= power factor of dielectric at frequency f.

For other conductor materials, the resistance of conductor of diameter d (and similarly for D) is

$$0.1(1/d)(f\mu_r\rho/\rho_{Cu})^{1/2} \text{ ohms/100 feet.}$$

Refer to section on "Skin Effect," in Chapter 6.

RESONANT LINES

Symbols

f_0= resonance frequency in megahertz
G_a= conductance load in mhos at voltage standing-wave maximum, equivalent to some or all of the actual loads
k= coefficient of coupling
n= integral number of quarter wavelengths
$p=k^2Q_{1s}Q_{2s}$= load transfer coefficient or matching factor
P_c= power converted into heat in resonator
P_m= power available from generator in watts $=E_{oc}^2/4R_{gen}$
P_x= power transferred when load is directly connected to generator (for single resonators); or an analogous hypothetical power (for two coupled resonators)
Q= figure of merit of a resonator as it exists, whether loaded or unloaded
Q_d= doubly loaded Q (all loads being included)
Q_s= singly loaded Q (all loads included except one). For a pair of coupled resonators, Q_{1s} is the value for the first resonator when isolated from the other. (Similarly for Q_{2s})
Q_u= unloaded Q
R_b= resistance load in ohms at voltage standing-wave minimum, equivalent to some or all of the actual loads
R_u= resistance similar to R_b except for unloaded resonator
R_1= generator resistance, referred to short-circuited end
R_2= load resistance
$S_x=R_1/R_2$ or R_2/R_1= mismatch factor between generator and load
Z_{10}= characteristic impedance of the first of a pair of resonators
θ_1= electrical angle from a voltage standing-wave minimum point

(A) Q of a resonator (electrical, mechanical or any other) is

$$Q=2\pi\frac{\text{(energy stored)}}{\text{(energy dissipated per cycle)}}$$

$$=2\pi f\frac{\text{(energy stored)}}{\text{(power dissipation)}}.$$

In a freely oscillating system, the amplitude decays exponentially.

$$I=I_0\exp(-\pi ft/Q).$$

(B) Unloaded Q of a resonant line:

$$Q_u=\beta/2\alpha$$

the line length being n quarter-wavelengths, where n is a small integer. The losses in the line are equivalent to those in a hypothetical resistor at the short-circuited end (E), p. 24-4:

$$R_u=n\pi Z_0/4Q_u.$$

(C) Loaded Q of a resonant line (Fig. 12):

$$Q^{-1}=Q_u^{-1}+(4R_b/n\pi Z_0)+(4G_a/n\pi Y_0)$$

$$=(4/n\pi Z_0)(R_u+R_b+G_a/Y_0^2).$$

All external loads can be referred to one end and represented by either R_b or G_a as in Fig. 13.

The total loading is the sum of all the individual loadings.

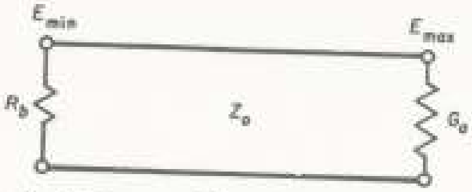


Fig. 12—Quarter-wave line with loadings at nominal short-circuit and open-circuit points.

General conditions:

$$R_b/Z_0 = G_a/Y_0 \ll 1.0$$

or, roughly, $Q > 5$.

(D) Input admittance and impedance: The converse of the equations for Fig. 13 can be used at the resonance frequency. Then R or G is the input impedance or admittance, while

$$R_b = n\pi Z_0/4Q_s$$

where Q_s = singly loaded Q with the losses and all the loads considered except that at the terminals where input R or G is being measured.

In the vicinity of the resonance frequency, the input admittance when looking into a line at a tap point θ_1 in Fig. 14 is approximately

$$Y = G + jB = \frac{n\pi Y_0}{4\sin^2\theta_1}\left(Q_s^{-1} + j2\frac{f-f_0}{f_0}\right)$$

provided

$$|f-f_0|/f_0 \ll 1.0$$

and

$$|[\theta(f-f_0)/f_0]\cot\theta_1| \ll 1.0$$

where $\theta = n\pi/2$ = length of line at f_0. It is not valid when $\theta_1 \approx 0, \pi, 2\pi$, etc., except that it is good near the short-circuited end when $f - f_0 \approx 0$.

Such a resonant line is approximately equivalent to a lumped LCG parallel circuit, where

$$\omega_0^2 L_1 C_1 = (2\pi f_0)^2 L_1 C_1 = 1.$$

Admittance of the equivalent circuit is

$$Y = G + j[\omega C_1 - (1/\omega L_1)]$$

$$\approx \omega_0 C_1 \{Q_s^{-1} + j2[(f-f_0)/f_0]\}.$$

Then, subject to the conditions stated above

$$L_1 = (4\sin^2\theta_1)/n\pi\omega_0 Y_0$$

$$C_1 = n\pi Y_0/(4\omega_0 \sin^2\theta_1) = nY_0/(8f_0\sin^2\theta_1)$$

$$G = n\pi Y_0/(4Q_s\sin^2\theta_1)$$

$$Q_s = \omega_0 C_1/G = 1/\omega_0 L_1 G.$$

Similarly, the input impedance at a point in series with the line (Fig. 13C and D) is

$$Z = R + jX = \frac{n\pi Z_0}{4\cos^2\theta_1}\left(Q_s^{-1} + j2\frac{f-f_0}{f_0}\right)$$

provided

$$|f-f_0|/f_0 \ll 1.0$$

and

$$|\theta[(f-f_0)/f_0]\tan\theta_1| \ll 1.0.$$

It is not valid when $\theta_1 \approx \pi/2, 3\pi/2$, etc.

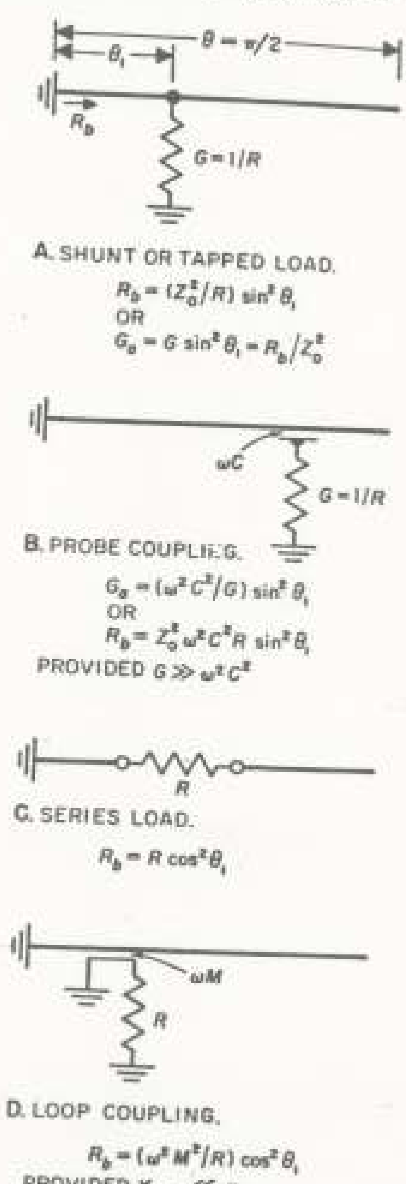


Fig. 13—Typical loaded quarter-wave sections with apparent R_b equivalent to the loading at distance θ_1 from voltage-minimum point of the line. Outer conductor not shown.

The voltage standing-wave ratio at resonance, on the generator (Fig. 15) is

$$S=(R_2+R_u)/R_1$$

$$=\frac{(R_2/R_1)Q_u+Q_d}{Q_u-Q_d}.$$

When $R_1=R_2$

$$S=\frac{1+Q_d/Q_u}{1-Q_d/Q_u}$$

$$\rho=Q_d/Q_u.$$

(E) Insertion loss (Fig. 15): At resonance, for either a distributed or a lumped-constant device

(dissipation loss)

$$=10\log_{10}(P_a/P_{out})$$
$$=20\log_{10}[1/(1-Q_d/Q_u)]$$
$$\approx 20\log_{10}(1+Q_d/Q_u)$$
$$\approx 8.7Q_d/Q_u \text{ decibels}$$

(mismatch loss)

$$=10\log_{10}(P_m/P_a)$$
$$=10\log_{10}[(1+S_a)^2/4S_a] \text{ decibels.}$$

The dissipation loss also includes a small additional mismatch loss due to the presence of the resonator. The error in the form $20\log_{10}(1+Q_d/Q_u)$ is about twice that of the form $8.7Q_d/Q_u$. The last expression ($8.7Q_d/Q_u$) is in error compared with the first, $20\log_{10}[1/(1-Q_d/Q_u)]$, by roughly $-50(Q_d/Q_u)$ percent for $(Q_d/Q_u)<0.2$.

The selectivity is given on page 9-7, where $Q=Q_d$. That equation is accurate over a smaller range of $(f-f_0)$ for a resonant line than it is for a single tuned circuit.

At resonance*

$$P_{out}/P_{in}=R_2/(R_u+R_2)$$

$$=\frac{Q_u-Q_d}{Q_u+(R_1/R_2)Q_d}=1-Q_s/Q_u$$

where Q_s is for the resonator loaded with R_2 only.

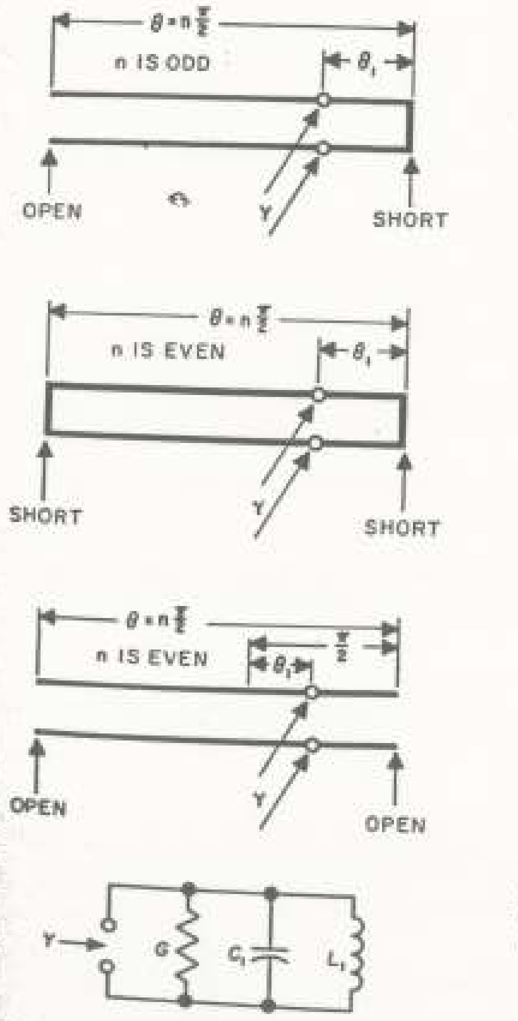


Fig. 14—Resonant transmission lines and their equivalent lumped circuit.

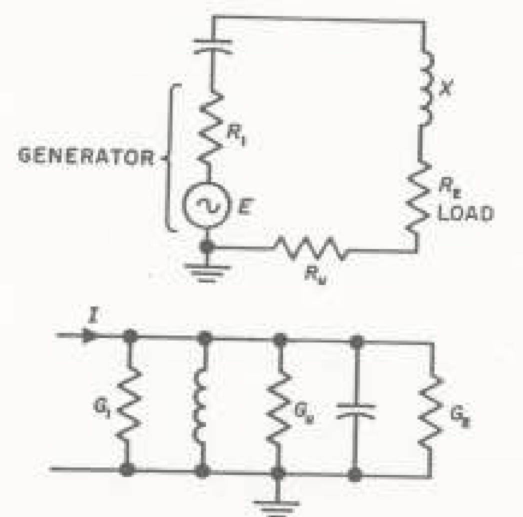


Fig. 15—Equivalent circuits of a resonant line (or a lumped tuned circuit) as seen at the short-circuited and open-circuited ends. All the power equations are good for either lumped or distributed parameters.

* When the line is resonated by a reactive load ($\theta\neq n\pi/2$), it is frequently preferable to use the resistance form of the equation. Compute R_u by the method given on p. 24-11, or on p. 24-5, where $Z_1=R_1(1-jB_1/G_0)$.

The maximum power transfer, for fixed Q_u, Q_d, and Z_0 occurs when $R_1=R_2$. Then

$$P_{out}/P_{in}=(Q_u-Q_d)/(Q_u+Q_d)=1-Q_s/Q_u$$

$$P_{out}/P_m=(1-Q_d/Q_u)^2$$

$$P_{in}/P_m=1-(Q_d/Q_u)^2.$$

When the generator R_1 or G_1 is negligibly small (then $Q=Q_s=Q_d$)

$$(P_{in}/P_{out})_s=Q_u/(Q_u-Q)$$

(F) Power dissipation ($=P_c$):

$$P_c/P_m=\frac{4(Q_d/Q_u)(1-Q_d/Q_u)}{1+R_2/R_1}.$$

For matched input and output ($R_1=R_2$)

$$P_c/P_m=2(Q_d/Q_u)(1-Q_d/Q_u)$$

$$\approx 2Q_d/Q_u \quad (\text{for } Q_d \ll Q_u)$$

$$P_c/P_{out}=2Q_d/(Q_u-Q_d)$$

$$P_c/P_{in}=2Q_d/(Q_u+Q_d).$$

For generator matched by load plus cavity

$$P_c/P_m=2Q_d/Q_u.$$

When the generator R_1 or G_1 is negligibly small

$$(P_c/P_{out})_s=Q/(Q_u-Q)$$

$$(P_c/P_{in})_s=Q_s/Q_u.$$

(G) Voltage and current:

At the current-maximum point of an n-quarter-wavelength resonant line

$$I_{sc}=4\left[\frac{P_mQ_d(1-Q_d/Q_u)}{(1+R_2/R_1)n\pi Z_0}\right]^{1/2}, \text{ rms amperes}$$

$$=4\left[\frac{P_mQ_d}{\{1+[(R_2+R_u)/R_1]\}n\pi Z_0}\right]^{1/2}.$$

When the generator R_1 or G_1 is negligibly small

$$I_{sc}=2\left[\frac{P_sQ_s}{n\pi Z_0(R_2+R_u)/R_s}\right]^{1/2}$$

where P_s = rated power of generator, R_s = rated load impedance as transformed into current-maximum point of cavity, and $I=I_{sc}\cos\theta_1$, while $E=Z_0I_{sc}\sin\theta_1$.

The voltage and current are in quadrature time phase.

When $R_1=R_2+R_u$ and $n=1$

$$I_{sc}\approx(8P_mQ_d/\pi Z_0)^{1/2}.$$

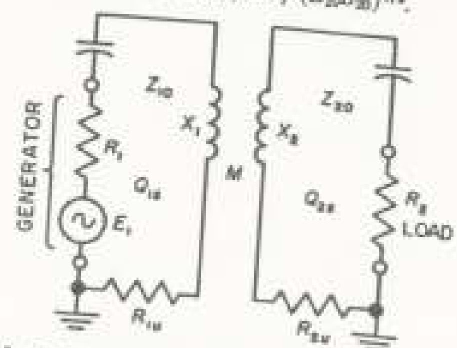
In a lumped-constant tuned circuit

$$I=2\left[\frac{P_mQ_d(1-Q_d/Q_u)}{(1+R_2/R_1)X}\right]^{1/2}.$$

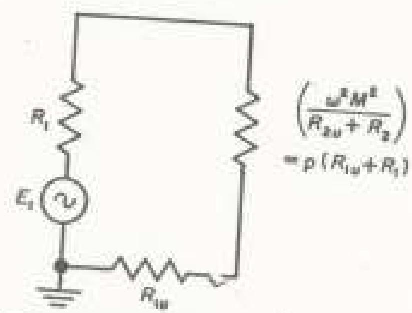
(H) Pair of coupled resonators (Fig. 16):

With inductive coupling near the short-circuited end of a pair of quarter-wave resonant lines

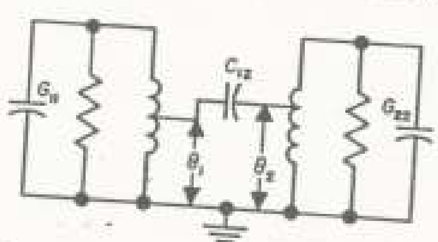
$$k=(4/\pi)\omega M/(Z_{10}Z_{20})^{1/2}.$$



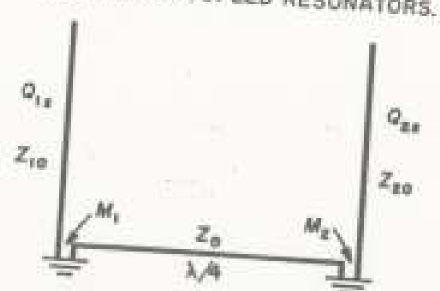
A. EQUIVALENT CIRCUIT WITH RESISTANCES AS SEEN AT THE SHORT-CIRCUITED END.



B. EQUIVALENT CIRCUIT OF FIRST RESONATOR AT RESONANCE FREQUENCY.



C. PROBE-COUPLED OR APERTURE-COUPLED RESONATORS.



D. QUARTER-WAVELENGTH LINE COUPLING.

Fig. 16—Two coupled resonators.